The Role of Theory

Pendulum Motion, Time Measurement, and the Shape of the Earth

Christian Huygens

Galileo's pendulum laws, developed in the early seventeenth century, were one of the foundations of modern science. Through their utilization in time measurement and hence in solving the longitude problem (how many degrees east or west of a given point a place is), they became a foundation of modern society. The laws were important for their insight into the behavior of swinging pendula. They were even more important because the methods of idealization and experimentation that were employed in their development became the characteristic methods of modern science. This is what differentiated the 'New Science' of the Scientific Revolution from the former observational and empiricist science of Aristotle that had for two centuries dominated Western efforts to understand nature. In a wonderful example of the surprises that science brings, the construction of an accurate pendulum-regulated clock enabled the oblate shape of the Earth to be ascertained 350 years before satellites were launched.

Christiaan Huygens (1629-1695) refined Galileo's pendulum laws and was the first to use these refined laws in creating a pendulum clock. Huygens stands out among the great scientific minds of the seventeenth century who addressed themselves to the improvement of time measurement and the solution of the longitude problem. Huygens possessed both manual and intellectual skills of the highest order. He was the son of a well-connected and wealthy Dutch diplomat, who, with good reason, called his son *mon Archimède.* Upon Huygens death, the great Leibniz wrote:

> The loss of the illustrious Monsieur Huygens is inestimable; few people knew him as well as I; in my opinion he equalled the reputation of Galileo and Descartes and aided theirs because he surpassed the discoveries that they made; in a word, he was one of the premier ornaments of our time.

Given that the 'times' contained Galileo, Descartes, Pascal, Boyle, Newton as well as Leibniz himself, this was obviously great praise.

Huygens' work falls, both temporally and conceptually, between that of Galileo (*Dialogue*, 1633, *Discourse* 1638) and Newton (*Principia* 1687, *Opticks* 1704). It was a bridge between the announcement and the fulfilment of the new science. His efforts embodied that combination of sophisticated mathematics and refined experimental technique that so characterised the new method of science introduced by Galileo and perfected by Newton the Galilean-Newtonian Paradigm.

CHRISTIAN HUYGENS

Huygens was born in The Hague in 1629. By age thirteen he had built himself a lathe, by seventeen he had independently ascertained Galileo's time-squared law of fall and Galileo's parabolic trajectory of a projectile, by twenty he had completed and published a study of hydrostatics, by twenty-three he formulated the laws of elastic collision, by twenty-five he was an optical lens grinder of national renown, by twenty-six, and using one of his own telescopes, he observed the ring of Saturn.

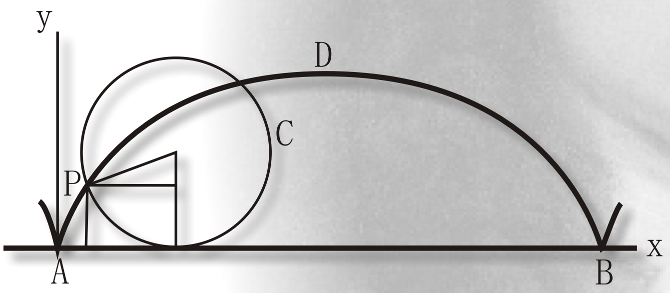
In 1663 he was made a member of England's newly founded (1662) Royal Society. In 1666, at age thirty-seven, he was invited by Louis XIV to be founding president of the *Académie Royal des Sciences*, an invitation he accepted, and a position he held, organizing its scientific affairs in a manner inspired by Roger Bacon's view of the scientific commonwealth, until 1681. Even during the bitter war between France and Holland that broke out in 1672 and that saw Huygens' father and brother occupying high positions in the opposing Court of William III of Orange, Louis XIV nevertheless retained Huygens as President of the *Académie*.

1. Scientific knowledge extends through time and place. In what way is science an individual pursuit, and in what ways is it also a communal and social pursuit?

Huygens changed two central features of Galileo's ideas, namely that the period of pendula varied with length, and that the isochronous curve of pendular motion was a circle. In contrast Huygens showed mathematically that period varied with the square root of length, and that the cycloid was the isochronous curve (Figure 1). Huygens thought so highly of the second result that he described it in a 1666 letter to Ismaël Boulliau as "the principal fruit that one could have hoped for from the science of accelerated motion, which Galileo had the honour of being the first to treat". This was high praise for something so seemingly obscure and esoteric. It is a reminder, given its utilization in producing truly isochronous pendulum motion in clocks, of the utility of 'theoretical' research.

FIGURE 1

Huygen's demonstration that period varies with the square root of length.



These ideas were both major theoretical breakthroughs. Galileo had argued that the circle was the tautochronous curve (i.e. the curved line, such that a heavy body, descending along it by the action of gravity, would always arrive at the lowest point in the same time, wherever in the curve it may begin to fall). Thus, an oscillating freely suspended pendulum would be isochronous because its bob moved in a circle. Huygens showed that it was not the circle, but the cycloid that was tautochronous and isochronous. He provides the following account of this insight:

> We have discovered a line whose curvature is marvelously and quite rationally suited to give the required equality to the pendulum. . . . This line is the path traced out in air by a nail which is fixed to the circumference of a rotating wheel which revolves continuously. The geometers of the present age have called this line a cycloid and have carefully investigated its many other properties. Of interest to us is what we have called the power of this line to measure time, which we found not by expecting this but only by following in the footsteps of geometry. (Huygens 1673/1986, p. 11)

2. Science is usually thought to be based primarily on observation. However, Galileo and Huygens (and later, Newton) based their ideas primarily on mathematics. Mathematical analysis and idealized conditions were two methodological changes that characterized the Scientific Revolution in the sixteenth and seventeenth centuries. Why would observation unlikely result in the idea of pendular isochronous curve motion?

Having shown mathematically that the cycloid was isochronous, Huygens then devised a simple way of making a suspended pendulum swing in a cycloidal path he made two metal cycloidal cheeks and caused the pendulum to swing between them. Huygens first pendulum clock was accurate to one minute per day; working with the best clockmakers, he soon made clocks accurate to one second per day.

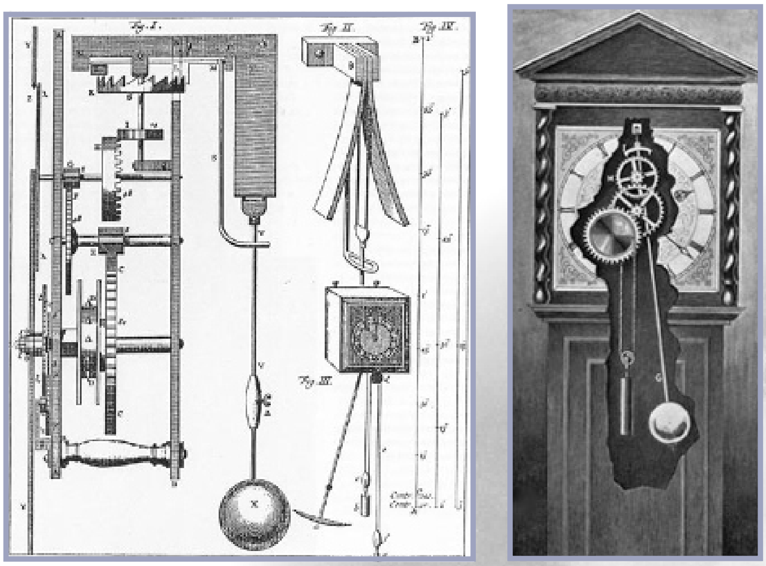
After showing that the period of a pendulum varied as the square root of its length, Huygens then derived the equation now familiar to all physics students:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

According to this equation, only the length of a pendulum affects its period. **π** is a constant, **g** remains constant provided the pendulum remains the same distance above or below sea level, and mass does not figure in the equation at all. So all pendula of a given length will have the same period, whether they be in France, England, Russia, Latin America, China or Australia. Huygens cleverly recognized the significance of this for solving the timekeeping and longitude problems. He had the additional insight that the pendulum could be used to solve an additional vexing problem, namely establishing a standard international unit of length. In 1673 he proposed that the international unit of length be the length of a pendulum whose period is two seconds.

Having an international unit of length, or even a national unit, would have been a major contribution to simplifying the chaotic state of measurement existing in science and everyday life. Within France, as in other countries, the unit of length varied from city to city, and even within cities. This was a significant problem for commerce, trade, construction, technology, and science. Many attempts had been made to simplify and unify the chaotic French system. The emperor Charlemagne in 789 issued an edict calling for a uniform system of weights and measures in France. One estimate is that in France alone there were 250,000 different local, measures of length, weight and

FIGURE 1



LEFT: Huygen's personal drawings for a clock, showing the cycloidal cheeks constraining the pendulum. RIGHT: Cut-away of a pendulum clock. The slowly falling weight provided small impulses to the pendulum to maintain its motion.

volume. The multiplicity of measures facilitated widespread fraud, or just smart business practice: merchants routinely bought according to 'long' measures and sold according to 'short' measures.

In the above formula, the length of a pendulum whose period is two seconds is easily determined to be one meter. This can be shown by taking any one meter long simple pendulum (a heavy nut on the end of a string suffices) and timing 10 or 20 swings – they will take 20 or 40 seconds. A great virtue of the two seconds pendulum as the international length standard was that it was a fully 'natural' standard. That is, it was something fixed by nature unlike standards based on the length of a king's arm or foot. And an international length standard would provide a related volume standard and hence a mass standard.

One advantage of the metric system is that units of length, volume and mass are related. For instance, 1cm^3 equals 1 mL and the mass of this much water is very close to 1 gram.

However one thing lying behind Huygen's proposal was that **g** be constant around the world (at least at sea level). This seemed a most reasonable assumption. Indeed to say that the Earth was not regular and spherical was tantamount to slandering the Creator. Surely God would not make a misshapen Earth. But in 1673, contrary to all expectation, the behavior of the pendulum did not reflect a constant **g**.

When Jean-Dominique Cassini (1625-1712) became director of the French A*cadémie Royale des Sciences* in 1669 he began sending expeditions into the different parts of the world to observe the longitudes of localities for the perfection of geography and navigation. The second such voyage was Jean Richer's to Cayenne in 1672-73. Cayenne was in French Guiana, at latitude approximately 5° N. It was chosen as a site for astronomical observations because equatorial observations were minimally affected by refraction of light passing through the Earth's atmosphere – the observer, the sun and the planets were all in the same plane.

The primary purpose of Richer's voyage was to ascertain the value of solar parallax and to correct the tables of refraction used by navigators and astronomers. A secondary consideration was checking the reliability of marine pendulum clocks which were being carried for the purpose of establishing Cayenne's exact longitude. Even by this second voyage scientific interests were being mixed with colonial ones.

The voyage was spectacularly successful in its primary purposes. But it was an unexpected consequence of Richer's voyage which destroyed Huygens' vision of a universal standard of length 'for all nations' and 'all ages.' Richer found that a meter long pendulum that had a period of two seconds at Paris, had to be shortened to exhibit the same period at Cayenne. Not much – 2.8 mm, about the thickness of a matchstick – but nevertheless shortened. Richer found that a Paris seconds-clock apparently lost 2½ minutes daily at Cayenne.

Newton acknowledged the veracity of Richer's claims, writing in his *Waste Book* of 1682, that:

> Monsr. Richer sent by y^e French King to make observations in the Isle of Cayenne (North Lat 5^{gr}) having before he went thither set his clocke exactly at Paris found there in Cayenne that it went too slow as every day to loose two minutes and a half for many days together and after his clock had stood & went again it lost 2½ minutes as before. Whence Mr Halley concludes that y^e pendulum was to be shortened in proportion of – to – to make y^e clock true at Cayenne. In Gorea y^e observation was less exact.

The fact that the weight of a body changed from place to place, as was manifest in the variation of the pendulum's period, sowed the seed for the conceptual distinction between weight and mass. The intuition was that although weight changed with change in gravity, nevertheless something about the 'massiness' of the body remained the same. Jean Bernoulli first introduced the distinction between mass and weight, and Newton, as will shortly be seen, clarified it by introducing the idea of inertial mass.

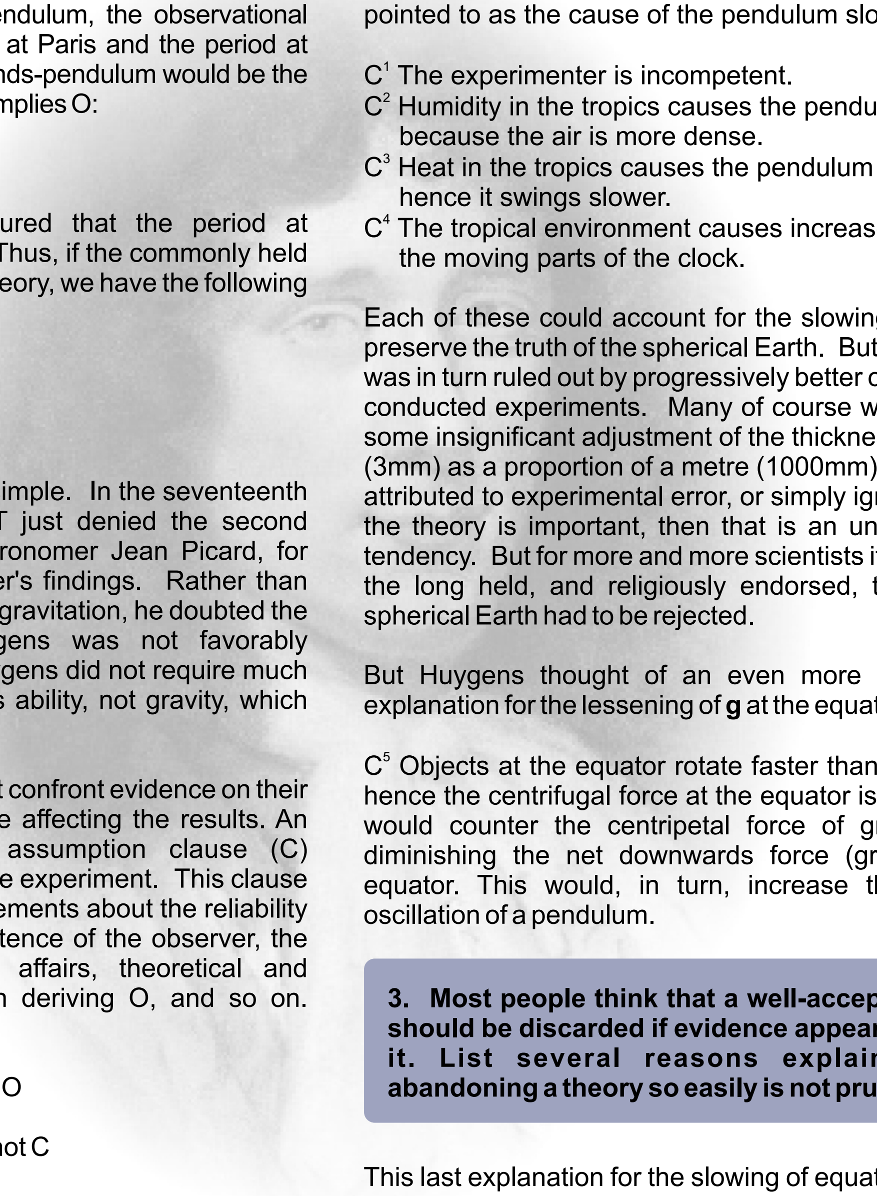
Richer's claim that the pendulum clock slows in equatorial regions nicely illustrates some key methodological matters about science, and about theory testing. The entrenched view since the second century BC was that the Earth was spherical (theory T). Assuming that gravity alone affects the period of a constant length pendulum, the observational implication was that the period at Paris and the period at Cayenne of Huygens' two seconds-pendulum would be the same (Observation O). Thus T implies O:

Premise 1: T → O

But Richer seemingly measured that the period at Cayenne was longer (not O). Thus, if the commonly held view that observation trumps theory, we have the following situation:

Premise 1: T → O
Premise 2: not O
therefore, not T

But theory testing is never so simple. In the seventeenth century, many upholders of T just denied the second premise of "not O." The astronomer Jean Picard, for instance, did not accept Richer's findings. Rather than accept the message of varying gravitation, he doubted the messenger. Similarly Huygens was not favorably disposed towards Richer. Huygens did not require much convincing that it was Richer's ability, not gravity, which was weak at Cayenne.



Others saw that theories did not confront evidence on their own, as other factors might be affecting the results. An 'other things being equal' assumption clause (C) accompanied the theory into the experiment. This clause characteristically included statements about the reliability of the instruments, the competence of the observer, the assumed empirical state of affairs, theoretical and mathematical devices used in deriving O, and so on. Thus:

Premise 1: T + C → O
Premise 2: not O
therefore, not T *or* not C

These people maintained confidence in T, and said that the assumption that other things were equal was mistaken. Humidity could have interfered with the pendulum swings, heat might have lengthened the pendulum, friction increased in the tropics, and so on. These, in principle, were legitimate concerns. But more and more evidence came in, and from other experimenters including Sir Edmund Halley, confirming Richer's observations. Thus "not O" became established and upholders of T, the spherical Earth theory, eventually had to adjust. This was not easy. Giving up established ideas in science is never easy, especially as the alternative theory was to accept that the Earth was not perfectly spherical, but rather was flattened at the poles and bulging at the equator.

To summarize, a number of obvious items in C could be pointed to as the cause of the pendulum slowing:

C^1 The experimenter is incompetent.
C^2 Humidity in the tropics causes the pendulum to slow because the air is more dense.
C^3 Heat in the tropics causes the pendulum to expand, hence it swings slower.
C^4 The tropical environment causes increased friction in the moving parts of the clock.

Each of these could account for the slowing, and hence preserve the truth of the spherical Earth. But each of them was in turn ruled out by progressively better controlled and conducted experiments. Many of course would say that some insignificant adjustment of the thickness of a match (3mm) as a proportion of a metre (1000mm) could just be attributed to experimental error, or simply ignored. And if the theory is important, then that is an understandable tendency. But for more and more scientists it seemed that the long held, and religiously endorsed, theory of the spherical Earth had to be rejected.

But Huygens thought of an even more sophisticated explanation for the lessening of **g** at the equator:

C^5 Objects at the equator rotate faster than at Paris and hence the centrifugal force at the equator is greater. This would counter the centripetal force of gravity, hence diminishing the net downwards force (gravity) at the equator. This would, in turn, increase the speed of oscillation of a pendulum.

3. Most people think that a well-accepted theory should be discarded if evidence appears to falsify it. List several reasons explaining why abandoning a theory so easily is not prudent.

This last explanation for the slowing of equatorial pendula was quite legitimate and appeared to save the spherical Earth theory. Many would be happy to just pick up this 'get out of jail free' card and continue to accept the spherical Earth theory. Huygens was not happy to do so. He calculated the actual centrifugal force at the equator. This was the exact opposite of the centripetal force, which was the force on an object rotating in a circle directed towards the centre.

When Huygens calculated this force, and took it from the expected equatorial force of gravity on the spherical Earth theory, he found that the diminished **g** did slow the pendulum but only to such a degree that a 1.5mm adjustment was required. This left 1.5mm of adjustment unaccounted for. This is not much more than the thickness of birthday card paper. But for such a small discrepancy, Huygens gave up the spherical Earth theory, and gave up his cherished vision of the two seconds pendulum acting as a universal standard of length. This was an imposing example of commitment to the scientific outlook: do your best to defend a theory, but when the counter evidence prevails, be prepared to give it up.

The problem is, how long must counter evidence prevail. In the nineteenth century scientists noted that observations of Uranus' orbit departed significantly from that predicted by Newton's gravitational law. While some scientists at the time speculated that the inverse-square law might not apply at the distance of Uranus, most scientists, noting the enormous success of the Newtonian framework in other affairs, expected the anomaly to be accounted for without abandoning or modifying Newton's law. In 1835, *years* after the anomaly in Uranus' orbit was first recognized, the return of Halley's comet sparked the idea that celestial bodies beyond Uranus might exert a force on the planet large enough to explain the planet's orbital discrepancy. This confidence, rather than seeing the anomaly as falsifying a well-supported idea, was key in the prediction and discovery of Neptune in 1846.

In 1738 Voltaire, a champion of the Newtonian science, wrote on the Richer episode, drawing attention to the problems of adjustment that scientists experienced:

> At last in 1672, Mr Richer, in a Voyage to Cayenna, near the Line, undertaken by Order of Lewis XIV under the protection of Colbert, the Father of all Arts; Richer, I say, among many Observations, found that the Pendulum of his Clock no longer made its Vibrations so frequently as in the Latitude of Paris, and that it was absolutely necessary to shorten it by a Line, that is, eleventh Part of our Inch, and about a Quarter more.
>
> Natural Philosophy and Geometry were not then, by far, so much cultivated as at present. Who could have believed that from this Remark, so trifling in Appearance, that from the Difference of the eleventh of our Inch, or thereabouts, could have sprung the greatest of physical Truths? It was found, at first, that Gravity must needs be less under the Equator, than in the Latitude of France, since Gravity alone occasions the Vibration of a Pendulum.
>
> In Consequence of this it was discovered, that, whereas the Gravity of Bodies is by so much the less powerful, as these Bodies are farther removed from the Centre of the Earth, the Region of the Equator must absolutely be much more elevated than that of France; and so must be farther removed from the Centre; and therefore, that the Earth could not be a Sphere.

Voltaire went on to say that:

> Many Philosophers, on occasion of these Discoveries, did what Men usually do, in Points concerning which it is requisite to change their Opinion; they opposed the new-discovered Truth.

4. (a) What does this story indicate about the complexity of testing scientific theories (i.e. consider what factors affect if and how quickly theories are abandoned, modified, and accepted)? (b) Why must an alternative theory exist before scientists will consider abandoning a previously accepted theory?

The Role of Theory: Pendulum Motion, Time Measurement, and the Shape of the Earth written by Michael R. Matthews, Michael P. Clough, & Craig Ogilvie.

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